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Technical note

Mitered offset for profile machining

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Abstract

In profile machining, it is very important to protect sharp vertices from being eroded. To meet this technological requirement, this paper proposes a mitered offsetting algorithm, which can be used for profiling tool path generation. The proposed algorithm is based on the pair-wise interference-detection (PWID) *offset algorithm* suggested by Choi and Park (Comput-Aided Des 31 (1999) 735). For mitered offsetting, we expand the PWID *offset algorithm* by developing a new method to remove global interference.

Keywords: Profile machining; Mitered offset; Tool path generation; Pair-wise interference-detection offset

1. Introduction

One of the most popular and important functions of CAD/CAM systems is generating tool paths for carving 2.5-dimensional (2.5D) parts, which can be described by strings of curves outlining the boundaries (profile curves) of the parts. The term 'profile machining' is used to refer to the milling vertical surfaces described by profile curves [1]. Profile machining requires higher precision (1/1000 mm) than regular 3D machining (1/100 mm) and especially the erosion of sharp vertices should be avoided.

Tool paths for profile machining have been generated by making use of conventional 2D curve offsetting algorithms. Fig. 1(a) shows an example of a conventional type tool path for profile machining. Observe that the sharp vertex P_1 is milled by an arc-tool-path-element, and it seems good enough. However, technologically it is inadequate to use this arc-tool-path-element, because the cutter contacts the sharp vertex (P_1) all along the arc. Due to the oscillations of the machine and the cutter, the sharpness of the corner is destroyed. Though most CNC controllers provide cutter radius compensation functionality with a mitered offset option, it is not applicable to the profile machining of curves with global interference, since the controller looks only a few segments ahead. As shown in Fig. 1(b), mitered offsetting can avoid the vertex erosion problem by replacing the arc with line segments. The concept of the mitered offset seems very suitable for profile machining, however, there has been little research.

The conventional offsetting problem has been widely studied because of its various applications in designing and manufacturing. In the literature, the point-sequence curve (PS-curve) offsetting problem has been approached from three different directions [2]: the pair-wise offset, Voronoi diagram, and pixel-based approaches. The pair-wise offset approach consists of four steps. (1) An offset segment is generated for each element of a boundary curve (input curve); (2) a raw offset-curve is constructed by closing the gaps with arcs; (3) all pair-wise self-intersections in the raw offset-curve are detected; and (4) all invalid loops are removed. Fig. 2 shows the basic geometric entities related to the pair-wise offset method. The thick curve denotes the original curve and the thin curve is its raw offset-curve. The raw offset-curve is divided into loops at its self-intersection points, and a loop that has to be removed to obtain valid offset curves is called an invalid loop. As shown in Fig. 2, there are two types of invalid loops: a local invalid loop bounded by a single self-intersection point and a global invalid loop bounded by a pair of self-intersection points. The pair-wise offset method is intuitive and simple. As pointed out by Held [5], however, the intersection-detection step (Step 3) is time consuming, and the loop-removal step (Step 4) is prone to numerical errors. The Voronoi diagram method is known to be more efficient and robust, but it may also suffer from numerical instability, such as near-circular singularity [2]. The strength of the pixel-based methods is robustness, but they require a large amount of memory as well as excessive computation time to

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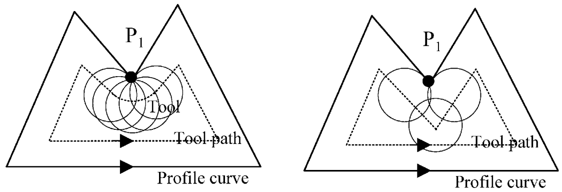
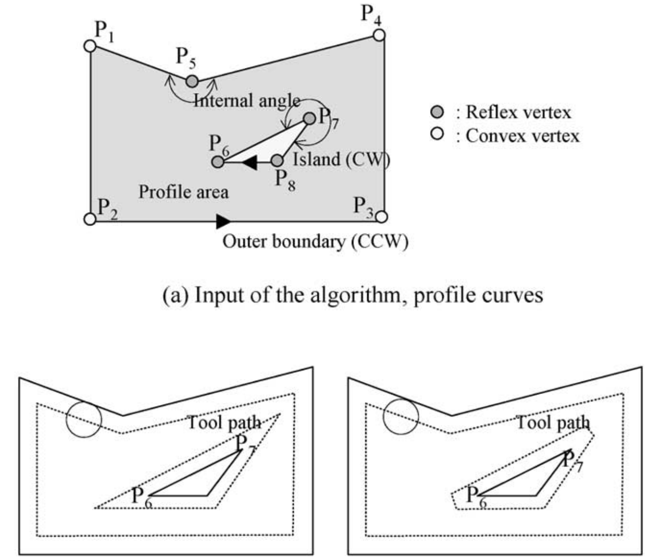


(a) Conventional offset (b) Mitered offset

Fig. 1. Conventional offset and mitered offset.



(a) Input of the algorithm, profile curves

(b) Undesirable tool path (c) Desirable tool path

Fig. 3. Input and output of the mitered offsetting algorithm.

achieve a desired level of precision. Recently, Choi and Park [2] proposed the pair-wise interference-detection (PWID) offset algorithm [2], to cope with the difficulties of the pair-wise offset approach. One of the salient features of the PWID offset algorithm is that it removes all local invalid loops before constructing a raw offset-curve by invoking a PWID test. As a result, the PWID offset algorithm avoids near-circular singularity causing numerical instability.

Our objective is to develop a mitered offsetting algorithm, which can handle an area including multiple islands with arbitrary shapes. Section 2 describes our approach to the mitered offsetting problem, and the proposed algorithm is explained in Section 3. Finally, concluding remarks are given in Section 4.

2. Approach to mitered offsetting

Before designing the mitered offset algorithm, we need to identify the functional requirements of the algorithm in terms of its input and output. There are three requirements of the algorithm: (1) the algorithm should be able to handle arbitrarily shaped profiles which may be an area including multiple islands, (2) the generated tool path should guarantee no gouge, and (3) the generated tool path should protect sharp vertices from being eroded. Fig. 3(a) shows an example of an input of the algorithm, which consists of two PS-curves. We assume that the orientations of the outer boundary and inner boundary (island) are counter-clockwise (CCW) and clockwise (CW), respectively. In the profile area, a vertex is called *reflex* if the internal angle between its incident segments is greater than π, and *convex* otherwise (Fig. 3(a)).

As mentioned earlier, the generated tool path should protect the reflex vertices (sharp vertices) from being eroded. Two examples of profiling tool paths are shown in Fig. 3(b) and (c), and both seem to satisfy the functional requirements, no gouging and no erosion of sharp vertices. However, machining operators would prefer the tool path in Fig. 3(c), because its length is shorter than that of the tool path in Fig. 3(b). To reduce unnecessary tool travel, we need to employ different methods for cutting reflex vertices according to their internal angles. As shown in Fig. 4, a reflex vertex having an internal angle larger than 1.5π can be machined efficiently by cutting out the unnecessary tool path.

One may think of two possible approaches to the mitered offsetting problem, one is based on the concept of the straight skeleton (similar with the Voronoi diagram) and the other is based on the PWID offsetting approach. A straight skeleton (angular bisector network, ABN) of a

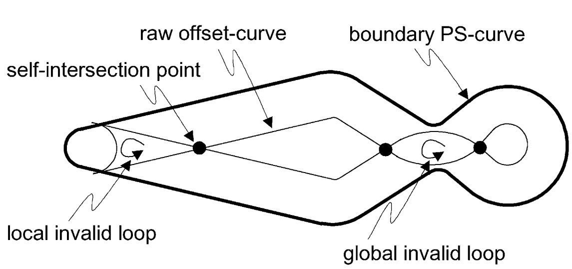
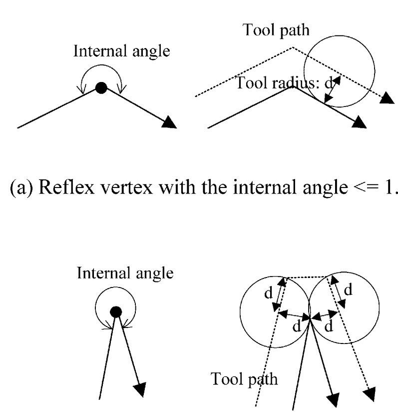


Fig. 2. Basic geometric entities in the pair-wise offset approach.



(a) Reflex vertex with the internal angle <= 1.5π

(b) Reflex vertex with the internal angle > 1.5π

Fig. 4. Cutting reflex vertices according to their internal angles.

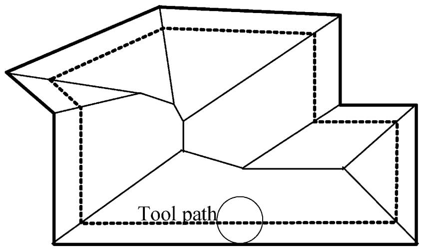


Fig. 5. Straight skeleton and mitered offsetting.

planar polygon, which can be grasped as a modification of a planar Voronoi diagram without parabolic arcs, has been successfully used by Oliva et al. [6] as a part of a system for 3D reconstruction of objects from a given set of 2D contours in parallel cross-sections. As shown in Fig. 5, it is possible to use the straight skeleton for the profiling tool path generation. The straight skeleton of the polygon is defined as the union of the pieces of angular bisectors traced out by polygon vertices during the shrinking process. Felkel and Obdrzalek [7] reported a detailed implementation procedure for the straight skeleton, and their experiment shows that the execution time of their algorithm works with quadratic time complexity.

In this article, we mainly focus on the development of a mitered offsetting algorithm based on the PWID offsetting approach. The simplest approach may be to replace the arcs of the offset curves with line segments. However, the replaced line segments may cause a global interference problem (gouge). To avoid the problem, we expand the PWID offset algorithm by developing a new method to remove global interference.

3. Mitered offsetting based on PWID offset

The main drawback of the original PWID offset algorithm is that it is applicable only to a PS-curve having no 'islands'. However, recently, the original algorithm has been expanded to offset areas including islands [3]. Before explaining the mitered offsetting procedure it may be helpful to summarize the PWID offset algorithm. To aid in the explanation, let us introduce some terms. A point on a PS-curve becomes an *interfering point* if its *tangential circle* intersects with the PS-curve (Fig. 6). A range of interfering

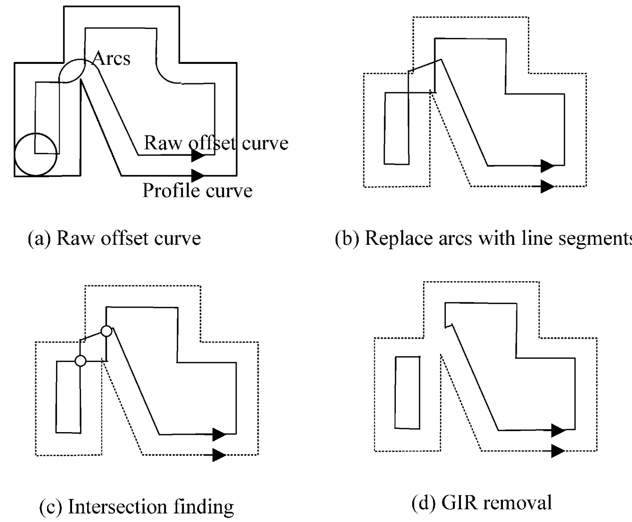


Fig. 7. Procedure of the suggested mitered offsetting algorithm.

points is called an *interfering-range*. An interfering-range corresponding to a local invalid loop is called a *local interfering-range* (LIR), and an interfering range belonging to a global invalid loop is called a *global interfering-range* (GIR).

One of the distinctive features of the PWID offset algorithm is that all LIRs are removed before constructing a raw offset-curve by invoking a PWID test. During the PWID test, each pair of elementary offset segments is tested for interference and then the *interfering segments* are successively removed. The resulting raw offset-curve, which now contains only GIRs (if any), is subjected to a sweep line algorithm [4] to find all self-intersections. If global invalid loops are detected, they are removed by invoking the PWID test again. The PWID test, which is the heart of the algorithm, is a function for removing LIRs and GIRs with convex vertices and self-intersection points, respectively. The overall procedure of the PWID offset algorithm can be described as follows:

1. Remove all LIRs by applying PWID tests with convex vertices on the PS-curve.
2. A raw offset-curve is constructed from the resulting boundary PS-curve.
3. All the pair-wise intersections in the raw offset-curve are detected.

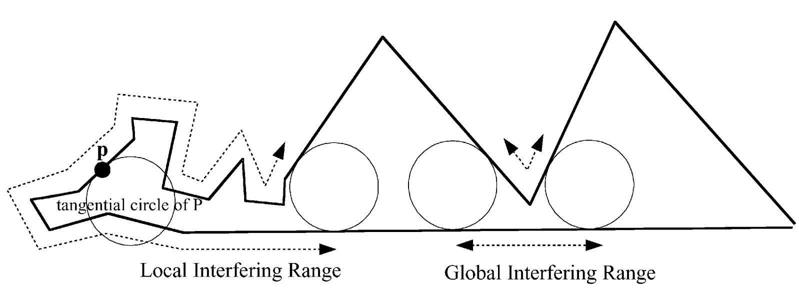


Fig. 6. Interfering point and interfering-range (LIR, GIR).

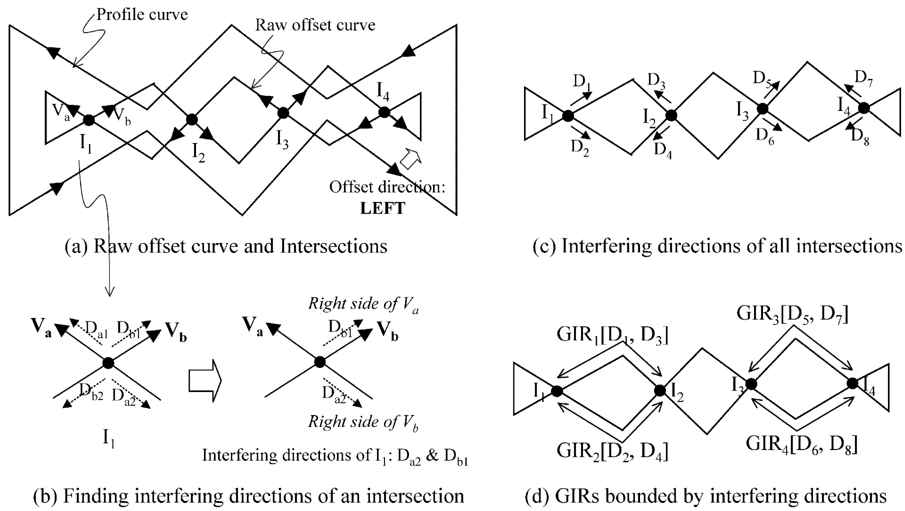


Fig. 8. New GIR removal procedure for mitered offsetting.

4. Remove all GIRs by applying PWID tests with the intersection points.

For mitered offsetting, we replace the arcs of the raw offset-curve (output of Step 2) with line segments, as shown in Fig. 7. We then have to develop a new GIR removal method, because Step 4 requires the PWID test, which is not compatible with mitered offsetting. First, we introduce the concept of 'interfering direction'. Fig. 8(a) shows a profile curve (CCW) and its raw offset-curve generated by offsetting the profile curve with 'LEFT' offset direction. LIRs already have been removed by the first step of the PWID offset algorithm. The raw offset-curve includes four intersections indicating the existence of GIRs. As shown in Fig. 8(b), an intersection (I_1) is caused by two intersecting line segments (V_a and V_b), and we can define four directions (D_{a1}, D_{a2}, D_{b1} and D_{b2}) from an intersection (I_1). Observe that if a tool follows D_{b1} (or D_{a2}) then it gouges the profile curve, and we call these directions interfering directions of I_1. Identifying two interfering directions among four candidate directions is quite simple. If the offset direction is LEFT (or RIGHT), then the candidate directions indicating the RIGHT side (or LEFT side, respectively) of the intersecting line segments become the interfering directions. For example, D_{b1} and D_{a2} are the interfering directions of I_1, because D_{b1} and D_{a2} lie on the RIGHT side of V_a and V_b, respectively. In this way, we can identify eight interfering directions, as shown in Fig. 8(c). Observe that a GIR has to be bounded by two interfering directions, i.e. we can easily extract all GIRs by tracing the raw offset-curve (Fig. 8(d)). After identifying all GIRs, we can get the valid offset curve by removing the union range of GIRs. Fig. 9 shows a more complicated example including 12 interfering directions (6 GIRs). Observe that the set of extracted GIRs may be different according to the processing order of interfering directions; however, the union of GIRs is invariant. The suggested GIR removal algorithm can be applied to offsetting areas including multiple islands in the same way.

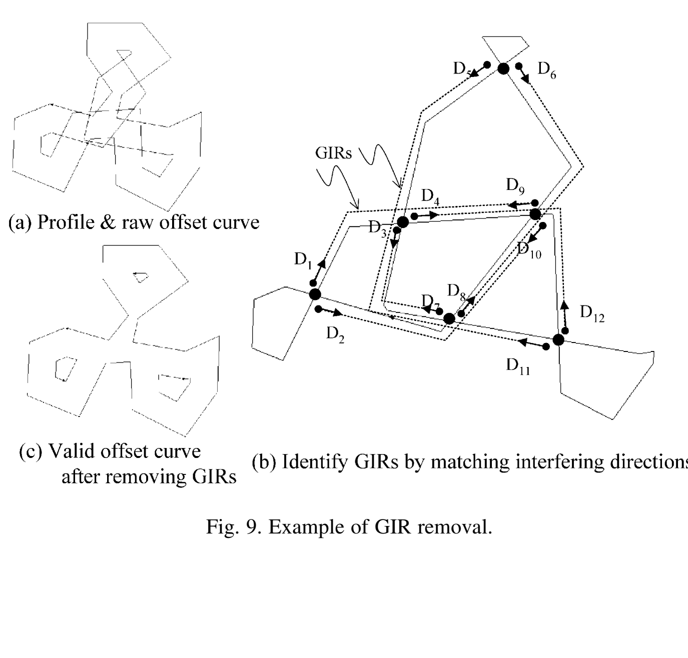


Fig. 9. Example of GIR removal.

4. Conclusions

For profile machining, a mitered offsetting algorithm is proposed. The generated tool path avoids the sharp vertex erosion problem, which is one of the most important technological requirements in profile machining. The mitered offsetting algorithm is developed by expanding the PWID offset algorithm [2], which is a conventional offsetting algorithm involving arcs. The original PWID offset algorithm consists of four steps: (1) LIR removal, (2) raw offset-curve construction, (3) intersection finding, and (4) GIR removal. For mitered offsetting, we replace the arcs of the raw offset-curve with line segments, and we develop a new GIR removal method, because Step 4 requires the PWID test, which is not compatible with mitered offsetting. The newly developed GIR removal method is much more efficient than that of the original PWID offset algorithm, because the new method identifies GIRs with a simple tracing operation.

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