Computational Creation of a New Illusionary Solid Sign

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***Abstract*—We present a computational method to create a new illusionary solid sign inspired by two kinds of illusions; "hollow mask illusion" and "crater illusion". The three-dimensional vertices of the illusionary solid sign are obtained by the *straight line Voronoi diagram* for a given base shape. We also provide a specific example of our new illusionary solid sign, so-called *hollow arrow sign*. Just like the name implies, our new sign has a hollow structure. However, we perceive the sign as a bulgy shape, when it is illuminated from below.**

***Keywords*-Computational Illusion; Straight line Voronoi diagram; Hollow mask illusion;**

I. Introduction

We see the world in three dimensions. However, the information from the external world is projected onto the two-dimensional retinal image, when we perceive the world through our eyes. We sometimes fall into an serious error, which is a kind of visual illusions, in the process of three-dimensional reconstruction due to the lack of depth information in two-dimensional retinal image.

Now, let us introduce a couple of powerful examples of depth inversion. One example is "hollow mask illusion" [1]–[6]. If a hollow mask of a face is viewed from a distance of a few feet even under dynamic binocular viewing conditions, the impression is of a normal face with the nose nearer to the observer than the forehead, that is, the mask appears as a convex face. Actually, the mask appears a concave face only at very close range, since the stereoscopic information dominates. Another example is "crater illusion" [7]–[11]. Although the craters are actually bumps as shown in the picture of crater [10], the alternate interpretation of the craters is occasionally preferred. Especially, this preference can be reinforced by turning the image upside down. The key of this illusion is shading effect. Shading indicates spatial changes of brightness of illuminated region, that is, the information obtained from the shading can be a cue of depth perception of the objects in the picture. In particular, the inference of shape from shading widely relies on the fact that light comes from above our heads, so-called "light-from-above" prior [12]. This kind of visual illusions due to light-from-above prior is provided by not only the picture of craters but also the shaded image of a planar surface which appears to be covered by bumps and dents [11] . Accordingly, the solid is generally perceived as a bump, when the upside of the solid is brighter than the downside. On the other hand, when the upside of the solid is darker than the downside, the solid is perceived as a dent [13], [14]. Thus, the shading plays an important role in the crater illusion. Whereas, the shading is not essential in the hollow mask illusion, since we have a prior knowledge that a face is a convex shape. Note, however, that the effect of the hollow mask illusion can be enhanced by the shading when the hollow mask is illuminated from below. In general these illusions are accidentally discovered or created on an empirical basis. However, anyone can create the intended illusionary solid sign by our method and there are no similar approaches has been presented before from the aspect of a creation of visual illusions.

In this contribution, we start to propose the general procedure for an illusionary solid sign to generate the hollow structure whose surface is completely visible from a wide range of view angles in Sec. II. Then, in Sec. III we provide a specific example of a new solid sign with the hollow structure, so-called *hollow arrow sign*, and demonstrate its visual illusion. Finally, Sec. IV is devoted to the concluding discussions.

II. Generation of the hollow structure with the same declination of walls

Let P be a polygon, i.e., a two-dimensional region bounded by a closed sequence of straight line segments.

We want to generate a hollow structure whose bottom coincides with P. The simplest hollow might be a cylindrical structure with the base P, that is, a hole generated by sweeping P vertically. However, the cylindrical hollow is not suitable for generating the hollow mask illusion, because the vertical walls of the hollow can be seen from only a narrow range of view angles. We need a hollow structure whose surface is completely visible from a wide range of view angles, because we want to see the hollow surface while we move through in front of the hollow structure.

In order to meet this requirement, we generate a hollow with slanted walls. In particular we generate all the walls with the same declination. For that purpose we employ the Voronoi diagram based on the straight skeleton proposed by Aichholzer et al. [15], and developed by many researchers (e.g., [16], [17]).

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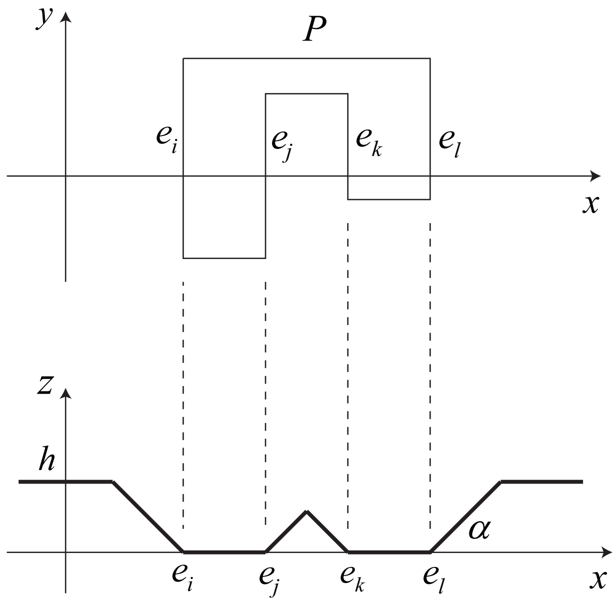

Figure 1. A polygon and a section of the associated hollow structure.

Let $(v_1, v_2, \ldots, v_n)$ be a counterclockwise sequence of the vertices of P, and let e_i be the straight line segment, called an *edge*, connecting v_i and v_{i+1} for $i = 1, 2, \ldots, n$, where v_{n+1} is read as v_1. We denote the set of the edges as $E_p = \{e_1, e_2, \ldots, e_n\}$.

Suppose that we fix the polygon P on the xy plane, and will construct the hollow structure with the height h. Fig. 1 shows our basic idea using a simple polygon. As shown in the upper part of this figure, we consider a simple polygon P as an example that has four edges e_i, e_j, e_k, e_l crossing the x axis perpendicularly. Let α be the angle between the xy plane and the slanted walls. Then, as shown in the lower part of Fig. 1, which is the section of the hollow structure with the xz plane, slanted walls are generated from the edges e_i and e_l so that the horizontal distance from the associated edges become $h \cot \alpha$. If the slanted wall intersects with another wall within the distance as in the case of e_j and e_k in Fig. 1, the wall ends at the intersection.

So our next question is: how shall we locate the wall region for each edge of the polygon as the xy plane ? To answer this question, we can employ the straight line Voronoi diagram in the two-dimensional plane.

Let l and l' be mutually parallel line segments. We define the *straight line distance* between l and l' as the distance between two lines (extended infinitely) respectively containing l and l'. Note that the straight line distance between l and l' is different from the Euclidean distance between l and l'; the Euclidean distance depends on whether the orthographic projection of l' onto the line containing l overlaps with l, whereas the straight line distance does not.

With each edge l_i of the polygon P, we associate a moving line segment $l_i(t)$, where t (≥ 0) is a time variable. It is initially at e_i (i.e., $l_i(0) = e_i$) and moves outward in the same speed s changing its length in such a way that the terminal points coincide with the terminal points of the neighbor line segments. The motion of $l_i(t)$ terminates when it collides with another moving line segment or it reach the straight line distance $h \cot \alpha$ from the initial position $l_i(0)$. Therefore, at each time $t \geq 0$, the collection $l_i(t),\ i = 1, 2, \ldots, n$ form a polygon.

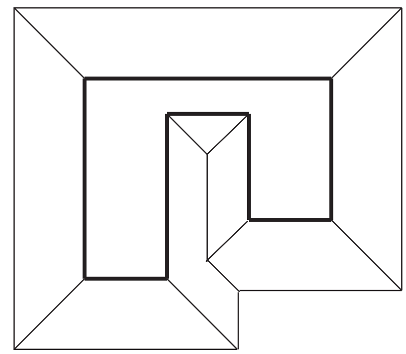

Figure 2. Straight line Voronoi diagram around the polygon shown in Fig. 1.

Suppose that the speed of the moving line segment $l_i(t)$ is $s = h \cot \alpha$. Then, $l_i(t)$ reaches the straight line distance $h \cot \alpha$ when $t = 1$ if it does not collide with another line segment.

For each e_i, let $R(e_i)$ denote the region swept by the line segment $l_i(t)$ for $0 \leq t \leq 1$. We call $R(e_i)$ the *Voronoi region of* e_i *with respect to the straight line distance*, or *the straight line Voronoi region* for short. We call the collection of the Voronoi regions of all the edges of P the *straight line Voronoi diagram* for P.

Fig. 2 show the straight line Voronoi diagram of the polygon in Fig. 1.

Suppose that we move the line segments $l_i(t),\ i = 1, 2, \ldots, n$, in the three-dimensional space, instead of the xy plane, in such a way that their height at time t is $z = ht$. Then, the swept surface is a planar slanted polygon with the angle α from the xy plane. Collecting those surfaces altogether, we get the walls of the hollow structure with the base P. We apply this method for constructing the hollow structure for a given polygon.

III. Hollow structure with the base of an arrow shape

Now, let us show a new illusionary solid sign "*hollow arrow sign*" in Fig. 3 whose base is an arrow shape as a specific example of the procedure to generate the hollow structure.

Here, the vertices $(v_1, v_2, \cdots, v_7)$ of the base P, which coincides with the surface of $z = 0$ (see Fig. 4 (a)) in the hollow arrow sign are given by $v_1 = (-7, 2, 0), v_2 =$

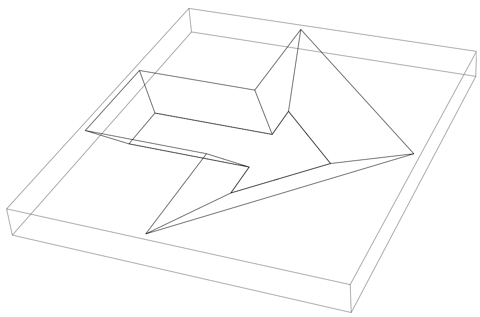

Figure 3. Overhead view of a new illusionary solid sign "*hollow arrow sign*". The base of this hollow structure is an arrow shape.

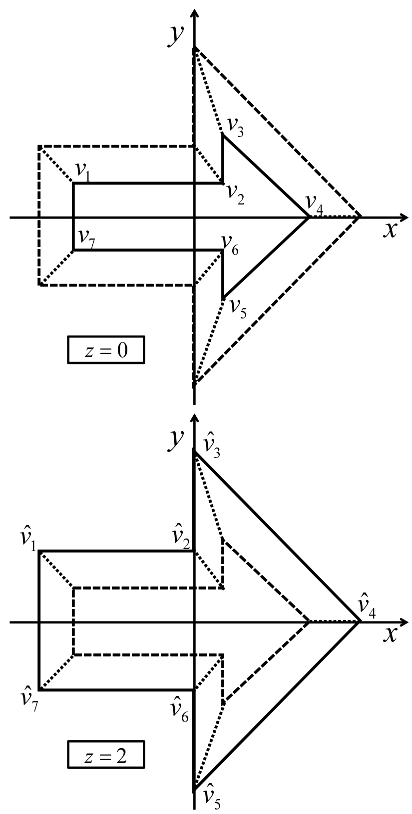

Figure 4. The labels of vertices on the surface of $z = 0$ (base P) and $z = 2$.

$(-7, -2, 0), v_3 = (2, -2, 0), v_4 = (2, -5, 0), v_5 = (7, 0, 0), v_6 = (2, 5, 0), v_7 = (2, 2, 0)$. The procedure to generate the hollow structure by straight line Voronoi diagram in the previous section, provides the set of vertices $(\hat{v}_1, \hat{v}_2, \cdots, \hat{v}_7)$ on the surface $z = 2$ (see Fig. 4 (b)) are $\hat{v}_1 = (-9, 4, 2), \hat{v}_2 = (-9, -4, 2), \hat{v}_3 = (0, -4, 2), \hat{v}_4 = (0, -9\sqrt{2}, 2), \hat{v}_5 = (9\sqrt{2}, 0, 2), \hat{v}_6 = (0, 9\sqrt{2}, 2), \hat{v}_7 = (0, 4, 2)$. One developed figure of the hollow arrow sign as shown in Fig. 5, is obtained via the development projection

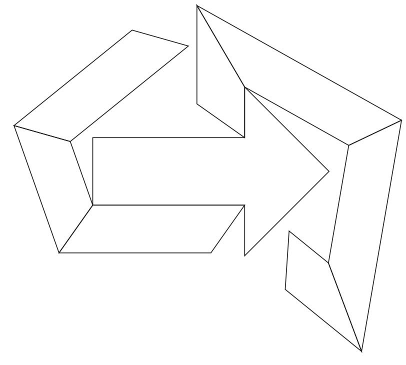

Figure 5. Developed figure of the hollow arrow shape.

from three-dimensional coordinates.

We actually make a three-dimensional hollow arrow sign from the developed figure printed on hard paper. Two snapshots of the same hollow arrow sign are shown in Fig. 6 and Fig. 7. When the hollow arrow sign is illuminated from above (in the case of Fig. 6), the shape of the arrow is perceived as a dent. Whereas, when the sign is illuminated from below (in the case of Fig. 7), the shape is perceived as a bump. In the case of Fig. 7, the hollow arrow sign creates a visual illusion in such a way that the depth is inversely perceived for one's eyes.

Thus, it is found that the shading effect also plays an important role in our illusionary sign from the difference of perceptions due to the illumination direction. Moreover, we have found that the hollow sign can appear to move in the same direction as the observer when the observer changes their observation point, since the surface of the hollow structure is completely visible from a wide range of view angles.

IV. Conclusion

In this contribution, we present a computational method to create a new illusionary solid sign with the hollow structure whose surface is completely visible from a wide range of view angles. This method built on the *straight line Voronoi diagram* for a given base shape provides the vertices of the illusionary solid sign. As a specific example of the illusionary solid sign, we demonstrate the *Hollow arrow sign*. Although the shape of our new sign definitely has a hollow structure, the new sign appears to a bulgy shape, when it is illuminated from below. Note that the variations of this hollow structure created by our method can be strongly applicable to the nameboards and signs as a new mode of expression. It is found that the shading effect plays an important role to perceive the depth in our illusionary sign from the results of different perceptions due to the illumination direction. In the near future we will extend our illusionary solid sign not only to the one with curved signs

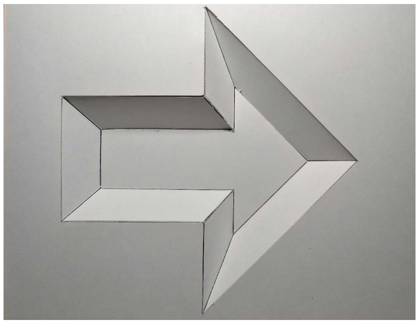

Figure 6. Hollow arrow sign illuminated from above.

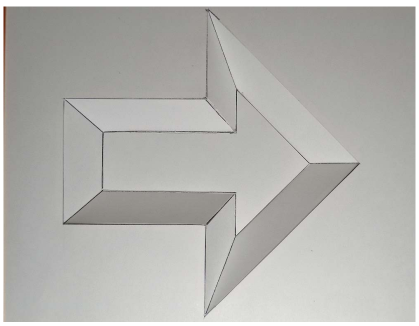

Figure 7. Hollow arrow sign illuminated from below.

which is a difficult problem to apply the straight line Voronoi diagram directly but also to the one with color to achieve the situation without the illumination on the lower side by calculating the appropriate shading color as if the sign is illuminated from below. This extension is meaningful for the practical application in terms of the reduction of facility cost.

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